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Hadron Masses From Novel Fat-Link Fermion Actions

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Abstract

The hadron mass spectrum is calculated in lattice QCD using a novel fat-link clover fermion action in which only the irrelevant operators in the fermion action are constructed using smeared links. The simulations are performed on a $16^3 \times 32$ lattice with a lattice spacing of $a = 0.125$ fm. We compare actions with $n = 4$ and 12 smearing sweeps with a smearing fraction of 0.7. The $n = 4$ Fat-Link Irrelevant Clover (FLIC) action provides scaling which is superior to mean-field improvement, and offers advantages over nonperturbative $\mathcal{O}(a)$ improvement, including a reduced exceptional configuration problem.

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I. INTRODUCTION

The origin of the masses of light hadrons represents one of the most fundamental challenges to the theory of strong interactions, Quantum Chromodynamics (QCD). Despite the universal acceptance of QCD as the basis from which to derive hadronic properties, there has been slow progress in understanding the generation of hadron mass from first principles. Solving the problem of the hadronic mass spectrum would allow considerable improvement in our understanding of the nonperturbative nature of QCD. The only available method at present to derive hadron masses directly from QCD is a numerical calculation on the lattice. In the last few years impressive progress has been made both in computer hardware and in developing more efficient algorithms, bringing realistic simulations of hadronic observables with sufficiently large volumes, small quark masses and fine enough lattices within reach.

The high computational cost required to perform accurate lattice calculations at small lattice spacings, however, has led to an increased interest in quark action improvement. In order to avoid the famous doubling problem, Wilson [1] originally introduced an irrelevant (energy) dimension-five operator (the "Wilson term") to the standard "naive" lattice fermion action, which explicitly breaks chiral symmetry at $\mathcal{O}(a)$. To extrapolate reliably to the continuum, simulations must be performed on fine lattices, which are therefore very computationally expensive. The scaling properties of this Wilson action at finite a can be improved by introducing any number of irrelevant operators of increasing dimension which vanish in the continuum limit.

The Sheikholeslami-Wohlert (clover) action [2] introduces an additional irrelevant dimension-five operator to the standard Wilson [1] quark action,

$$S_{\rm SW} = S_{\rm W} - \frac{iaC_{\rm SW}r}{4}\ \bar\psi(x)\sigma_{\mu\nu}F_{\mu\nu}\psi(x) \ , \tag{1}$$

where $S_{\rm W}$ is the standard Wilson action,

$$S_{\rm W} = \bar\psi(x)\left[\sum_\mu\left(\gamma_\mu\nabla_\mu - \frac{1}{2}ra\Delta_\mu\right) + m\right]\psi(x) \ , \tag{2}$$

∇_μ and Δ_μ are the standard covariant first and second order lattice derivatives,

$$\begin{aligned}\nabla_\mu\psi(x) &= \frac{1}{2a}\left[U_\mu(x)\psi(x+\mu) - U^\dagger_\mu(x-\mu)\psi(x-\mu)\right] ,\\ \Delta_\mu\psi(x) &= \frac{1}{a^2}\left[U_\mu(x)\psi(x+\mu) + U^\dagger_\mu(x-\mu)\psi(x-\mu) - 2\psi(x)\right] ,\end{aligned}$$

and $C_{\rm SW}$ is the clover coefficient which can be tuned to remove $\mathcal{O}(a)$ artifacts,

$$C_{\rm SW} = \begin{cases} 1 & \text{at tree-level} \ , \\ 1/u_0^3 & \text{mean-field improved} \ , \end{cases} \tag{3}$$

where u_0 is the tadpole improvement factor which corrects for the quantum renormalization of the operators. Nonperturbative (NP) $\mathcal{O}(a)$ improvement [3] tunes $C_{\rm SW}$ to all powers in g^2 and displays excellent scaling, as shown by Edwards *et al.* [4], who studied the scaling properties of the nucleon and vector meson masses for various lattice spacings (see also Section IV below). In particular, the linear behavior of the NP-improved clover actions, when plotted against a^2, demonstrates that $\mathcal{O}(a)$ errors are removed. It was also found in Ref. [4] that a linear extrapolation of the mean-field improved data fails, indicating that $\mathcal{O}(a)$ errors are still present.

A drawback to the clover action, however, is the associated problem of exceptional configurations, where the quark propagator encounters singular behavior as the quark mass becomes small. In practice, this prevents the use of coarse lattices ($\beta \gtrsim 5.7 \sim a \lesssim 0.18$ fm) [5,6]. Furthermore, the plaquette version of $F_{\mu\nu}$, which is commonly used in Eq. (1), has large $\mathcal{O}(a^2)$ errors, which can lead to errors of the order of 10% in the topological charge even on very smooth configurations [7].

The idea of using fat links in fermion actions was first explored by the MIT group [8] and more recently has been studied by DeGrand *et al.* [6,9], who showed that the exceptional configuration problem can be overcome by using a fat-link (FL) clover action. Moreover, the renormalization of the coefficients of action improvement terms are small. A drawback to conventional fat-link techniques, however, is that in smearing the links one leaves the SU(3) gauge group and must subsequently project back to SU(3), thus losing a rigorous connection to continuum QCD. It is because of this projection that APE smearing is only approximately gauge covariant. In addition, gluon interactions are removed at the scale of the cutoff. While this has some tremendous benefits, the short-distance quark interactions are lost. As a result decay constants, which are sensitive to the wavefunction at the origin, are suppressed.

A solution to these problems is to work with two sets of links in the fermion action. In the relevant dimension-four operators, one works with the untouched links generated via Monte Carlo methods, while the smeared fat links are introduced only in the higher dimension irrelevant operators. In this way the continuum limit of the theory is perfectly well defined.

In this paper we present the first results of simulations of the spectrum of light mesons and baryons using this variation on the clover action. In particular, we will start with the standard clover action and replace the links in the irrelevant operators with APE smeared [10], or fat links. We shall refer to this action as the Fat-Link Irrelevant Clover (FLIC) action. To the best of our knowledge, this is the first report of lattice QCD calculations using this novel fermion action.

In Section II we outline the details of our lattice simulations. Section III contains the procedure for creating the FLIC fermion action, and our results are presented in Section IV. Finally, in Section V we draw some conclusions and discuss future work.

II. THE GAUGE ACTION

The simulations are performed using a mean-field improved, plaquette plus rectangle, gauge action on a $16^3 \times 32$ lattice at $\beta = 6/g^2 = 4.60$, providing a lattice spacing $a = 0.125(2)$ fm determined from the string tension with $\sqrt{\sigma} = 440$ MeV. The tree-level $\mathcal{O}(a^2)$–Symanzik-improved gauge action [11] is defined as

$$S_{\mathrm{G}} = \frac{5\beta}{3} \sum_{\mathrm{sq}} \mathcal{R}e\mathrm{tr}(1 - U_{\mathrm{sq}}(x)) - \frac{\beta}{12u_0^2} \sum_{\mathrm{rect}} \mathcal{R}e\mathrm{tr}(1 - U_{\mathrm{rect}}(x)) \; , \tag{4}$$

where the operators $U_{\rm sq}(x)$ and $U_{\rm rect}(x)$ are defined as

$$\begin{aligned} U_{\rm sq}(x) &= U_\mu(x)U_\nu(x+\hat\mu)U^\dagger_\mu(x+\hat\nu)U^\dagger_\nu(x)\ , \qquad (5a)\\ U_{\rm rect}(x) &= U_\mu(x)U_\nu(x+\hat\mu)U_\nu(x+\hat\nu+\hat\mu)\\ &\quad \times U^\dagger_\mu(x+2\hat\nu)U^\dagger_\nu(x+\hat\nu)U^\dagger_\nu(x)\\ &+ U_\mu(x)U_\mu(x+\hat\mu)U_\nu(x+2\hat\mu)\\ &\quad \times U^\dagger_\mu(x+\hat\mu+\hat\nu)U^\dagger_\mu(x+\hat\nu)U^\dagger_\nu(x)\ . \qquad (5b)\end{aligned}$$

The link product $U_{\rm rect}(x)$ denotes the rectangular 1×2 and 2×1 plaquettes, and for the tadpole improvement factor we employ the plaquette measure

$$u_0 = \left(\frac{1}{3}\mathcal{R}e\,{\rm tr}\langle U_{\rm sq}\rangle\right)^{1/4}\ . \qquad (6)$$

Gauge configurations are generated using the Cabibbo-Marinari pseudoheat-bath algorithm with three diagonal SU(2) subgroups looped over twice. Simulations are performed using a parallel algorithm with appropriate link partitioning [12]. A total of 50 configurations are used in this analysis, and the error analysis is performed by a third-order, single-elimination jackknife, with the χ^2 per degree of freedom ($N_{\rm DF}$) obtained via covariance matrix fits.

III. FAT-LINK IRRELEVANT FERMION ACTION

Fat links [6,9] are created by averaging or smearing links on the lattice with their nearest neighbours in an approximately gauge covariant manner (APE smearing). The smearing procedure [10] replaces a link, $U_\mu(x)$, with a sum of the link and α times its staples

$$\begin{aligned} U_\mu(x)\ \rightarrow\ U^{\rm FL}_\mu(x)\ &=\ (1-\alpha)U_\mu(x) + \frac{\alpha}{6}\sum_{\substack{\nu=1\\ \nu\neq\mu}}^{4}\Big[U_\nu(x)U_\mu(x+\nu a)U^\dagger_\nu(x+\mu a)\\ &\qquad + U^\dagger_\nu(x-\nu a)U_\mu(x-\nu a)U_\nu(x-\nu a+\mu a)\Big]\,. \qquad (7)\end{aligned}$$

The mean-field improved FLIC action now becomes

$$S^{\rm FL}_{\rm SW} = S^{\rm FL}_{\rm W} - \frac{iC_{\rm SW}\kappa r}{2(u_0^{\rm FL})^4}\ \bar\psi(x)\sigma_{\mu\nu}F_{\mu\nu}\psi(x)\ , \qquad (8)$$

where $F_{\mu\nu}$ is constructed using fat links, and where the mean-field improved Fat-Link Irrelevant Wilson action is

$$\begin{aligned} S^{\rm FL}_{\rm W} = \sum_x \bar\psi(x)\psi(x) + \kappa\sum_{x,\mu}\bar\psi(x)\Big[\gamma_\mu\Big(\frac{U_\mu(x)}{u_0}\psi(x+\hat\mu) - \frac{U^\dagger_\mu(x-\hat\mu)}{u_0}\psi(x-\hat\mu)\Big)\\ - r\Big(\frac{U^{\rm FL}_\mu(x)}{u_0^{\rm FL}}\psi(x+\hat\mu) + \frac{U^{\rm FL\dagger}_\mu(x-\hat\mu)}{u_0^{\rm FL}}\psi(x-\hat\mu)\Big)\Big]\ , \qquad (9)\end{aligned}$$

and $\kappa = 1/(2m+8r)$. We take the standard value $r=1$. Our notation uses the Pauli representation of the Dirac γ-matrices defined in Appendix B of Sakurai [13]. In particular, the γ-matrices are hermitian and $\sigma_{\mu\nu} = [\gamma_\mu,\ \gamma_\nu]/(2i)$.

n	$u_0^{\rm FL}$	$(u_0^{\rm FL})^4$
0	0.88894473	0.62445197
4	0.99658530	0.98641100
12	0.99927343	0.99709689

TABLE I. The value of the mean link for different numbers of smearing sweeps, n, resulting from our simulations on a $16^3 \times 32$ lattice with a mean-field, plaquette + rectangle gluon action at $\beta = 6/g^2 = 4.60$, corresponding to $a = 0.125(2)$ fm.

As reported in Table I, the mean-field improvement parameter for the fat links is very close to 1. Hence, the mean-field improved coefficient for $C_{\rm SW}$ is expected to be adequate[1]. In addition, actions with many irrelevant operators (*e.g.* the D_{234} action) can now be handled with confidence as tree-level knowledge of the improvement coefficients should be sufficient. Another advantage is that one can now use highly improved definitions of $F_{\mu\nu}$ (involving terms up to u_0^{12}), which give impressive near-integer results for the topological charge [14].

In particular, we employ an $\mathcal{O}(a^4)$ improved definition of $F_{\mu\nu}$ in which the standard clover-sum of four 1×1 Wilson loops lying in the μ, ν plane is combined with 2×2 and 3×3 Wilson loop clovers. Bilson-Thompson *et al.* [14] find

$$F_{\mu\nu} = \frac{-i}{8}\left[\left(\frac{3}{2}W^{1\times1} - \frac{3}{20u_0^4}W^{2\times2} + \frac{1}{90u_0^8}W^{3\times3}\right) - \text{h.c.}\right] - \frac{1}{3}\text{tr}F_{\mu\nu} \qquad (10)$$

where $W^{n\times n}$ is the clover-sum of four $n \times n$ Wilson loops and where $F_{\mu\nu}$ is made traceless by subtracting 1/3 of the trace from each diagonal element of the 3×3 colour matrix. This definition reproduces the continuum limit with $\mathcal{O}(a^6)$ errors. On approximately self-dual configurations, this operator produces integer topological charge to better than 4 parts in 10^4.

Work by DeForcrand *et al.* [15] suggests that 7 cooling sweeps are required to approach topological charge within 1% of integer value. This is approximately 16 APE smearing sweeps at $\alpha = 0.7$ [16]. However, achieving integer topological charge is not necessary for the purposes of studying hadron masses, as has been well established. To reach integer topological charge, even with improved definitions of the topological charge operator, requires significant smoothing and associated loss of short-distance information. Instead, we regard this as an upper limit on the number of smearing sweeps.

Using unimproved gauge fields and an unimproved topological charge operator, Bonnet *et al.* [7] found that the topological charge settles down after about 10 sweeps of APE smearing at $\alpha = 0.7$. Consequently, we create fat links with APE smearing parameters $n = 12$

[1] Our experience with topological charge operators suggests that it is advantageous to include u_0 factors, even as they approach 1.

Wilson		FLIC ($n = 12$)		FLIC ($n = 4$)	
m_q (MeV)	iter	m_q (MeV)	iter	m_q (MeV)	iter
186	70	194	70	193	50
155	60	166	50	163	35
125	70	133	65	129	45
96	100	105	100	100	55
66	130	73	130	66	100

TABLE II. Quark masses and the corresponding average number of stabilised biconjugate gradient iterations for both Wilson and FLIC actions.

and $\alpha = 0.7$. This corresponds to ~ 2.5 times the smearing used in Refs. [6,9]. Further investigation reveals that improved gauge fields with a small lattice spacing ($a = 0.125$ fm) are smooth after only 4 sweeps. Hence, we perform calculations with 4 sweeps of smearing at $\alpha = 0.7$ and consider $n = 12$ as a second reference. Table I lists the values of $u_0^{\rm FL}$ for $n = 0$, 4 and 12 smearing sweeps.

A fixed boundary condition is used for the fermions by setting

$$U_t(\vec{x}, nt) = 0 \qquad \text{and} \qquad U_t^{\rm FL}(\vec{x}, nt) = 0 \qquad \forall\ \vec{x} \tag{11}$$

in the hopping terms of the fermion action. The fermion source is centered at the space-time location $(x, y, z, t) = (1, 1, 1, 3)$, which allows for two steps backward in time without loss of signal. Gauge-invariant gaussian smearing in the spatial dimensions is applied at the source to increase the overlap of the interpolating operators with the ground states.

IV. RESULTS

Hadron masses are extracted from the Euclidean time dependence of the calculated two-point correlation functions. For baryons, the correlation functions are given by

$$\langle G(t; \vec{p}, \Gamma)\rangle = \sum_x e^{-i\,\vec{p}\cdot\vec{x}}\ \Gamma^{\beta\alpha}\langle\Omega|T[\chi^\alpha(x)\bar{\chi}^\beta(0)]|\Omega\rangle\ , \tag{12}$$

where χ are standard baryon interpolating fields, Ω represents the QCD vacuum, Γ is a 4×4 matrix in Dirac space and α, β are Dirac indices. At large Euclidean times,

$$\langle G(t; \vec{p}, \Gamma)\rangle \simeq \frac{Z^2}{2E_p}\ e^{-E_p t}\ {\rm tr}\left[\Gamma(-i\gamma\cdot p + M)\right]\ , \tag{13}$$

where Z represents the coupling strength of $\chi(0)$ to the baryon, and $E_p = (\vec{p}^{\,2} + M^2)^{1/2}$. Selecting $\vec{p} = 0$ and $\Gamma = (1 + \gamma_4)/4$, the effective baryon mass is then given by

$$M(t + 1/2) = \log[G(t)] - \log[G(t + 1)]\ . \tag{14}$$

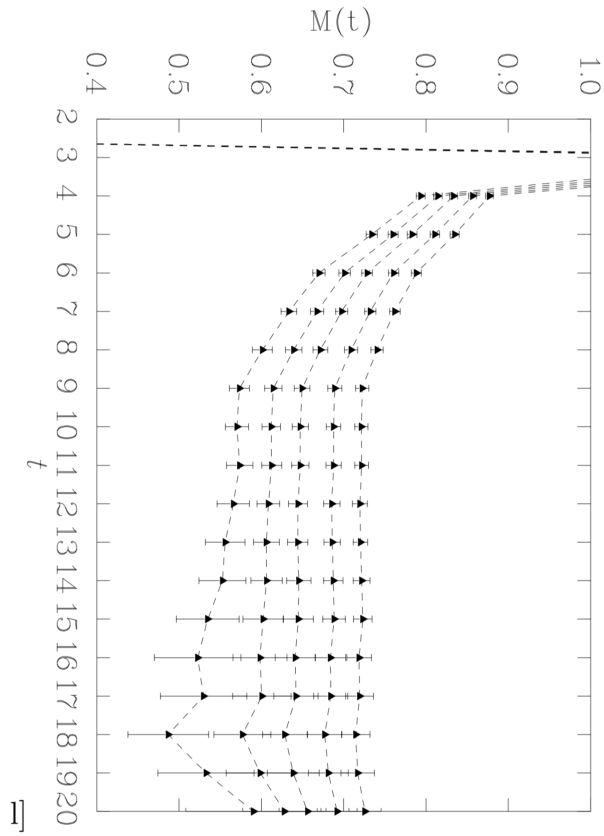

FIG. 1. Effective mass plot for the ρ meson for the FLIC action with 4 sweeps of smearing at $\alpha = 0.7$ from 50 configurations. The five quark masses decrease by value from top down and are listed in Table II.

Meson masses are determined via analogous standard procedures. The critical value of κ, κ_c, is determined by linearly extrapolating m_π^2 as a function of m_q to zero. The quark masses defined by $m_q = (1/\kappa - 1/\kappa_c)/(2a)$ are summarised in Table II, which also includes the mean number of Stabilised Biconjugate-Gradient (BiCGStab) [17] iterations required to invert the fermion matrix. The strange quark mass was taken to be the second heaviest quark mass in each case.

Figure 1 shows the ρ meson effective mass plot for the FLIC action when 4 APE smearing sweeps at $\alpha = 0.7$ are performed on the fat links ("FLIC4"). The effective mass plots for the other hadrons are similar, and all display acceptable plateau behavior. Good values of $\chi^2/N_{\rm DF}$ are obtained for many different time-fitting intervals as long as one fits after time slice 8. All fits for this action are therefore performed on time slices 9 through 14. For the Wilson action and the FLIC action with $n = 12$ ("FLIC12") the fitting regimes used are 9-13 and 9-14, respectively.

The behavior of the ρ, nucleon and Δ masses as a function of squared pion mass is shown in Fig. 2 for the various actions. The first feature to note is the excellent agreement between the FLIC4 and FLIC12 actions. On the other hand, the Wilson action appears

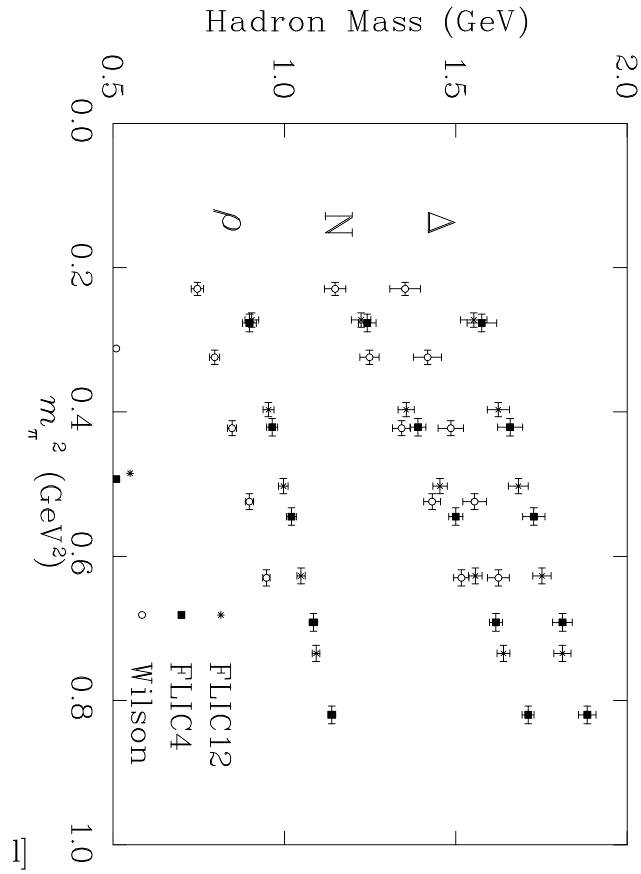

FIG. 2. Masses of the nucleon, Δ and ρ meson versus m_π^2 for the FLIC4, FLIC12 and Wilson actions.

to lie somewhat low in comparison. It is also reassuring that all actions give the correct mass ordering in the spectrum. The value of the squared pion mass at $m_\pi/m_\rho = 0.7$ is plotted on the abscissa for the three actions as a reference point. This point is chosen in order to allow comparison of different results by interpolating them to a common value of $m_\pi/m_\rho = 0.7$, rather than extrapolating them to smaller quark masses, which is subject to larger systematic and statistical uncertainties.

The scaling behavior of the different actions is illustrated in Fig. 3. The present results for the Wilson action agree with those of Ref. [4]. The first feature to observe in Fig. 3 is that actions with fat-link irrelevant operators perform extremely well. For both the vector meson and the nucleon, the FLIC4 action performs systematically better than the FLIC12. This suggests that 12 smearing sweeps removes too much short-distance information from the gauge-field configurations. On the other hand, 4 sweeps of smearing combined with our $\mathcal{O}(a^4)$ improved $F_{\mu\nu}$ provides excellent results, without the fine tuning of $C_{\rm SW}$ in the NP improvement program. Notice that for the ρ meson, a linear extrapolation of the mean-field improved clover points in Fig. 3 indicates that there is better improvement when using fat links in the irrelevant operators. While there are no NP-improved clover plus improved glue

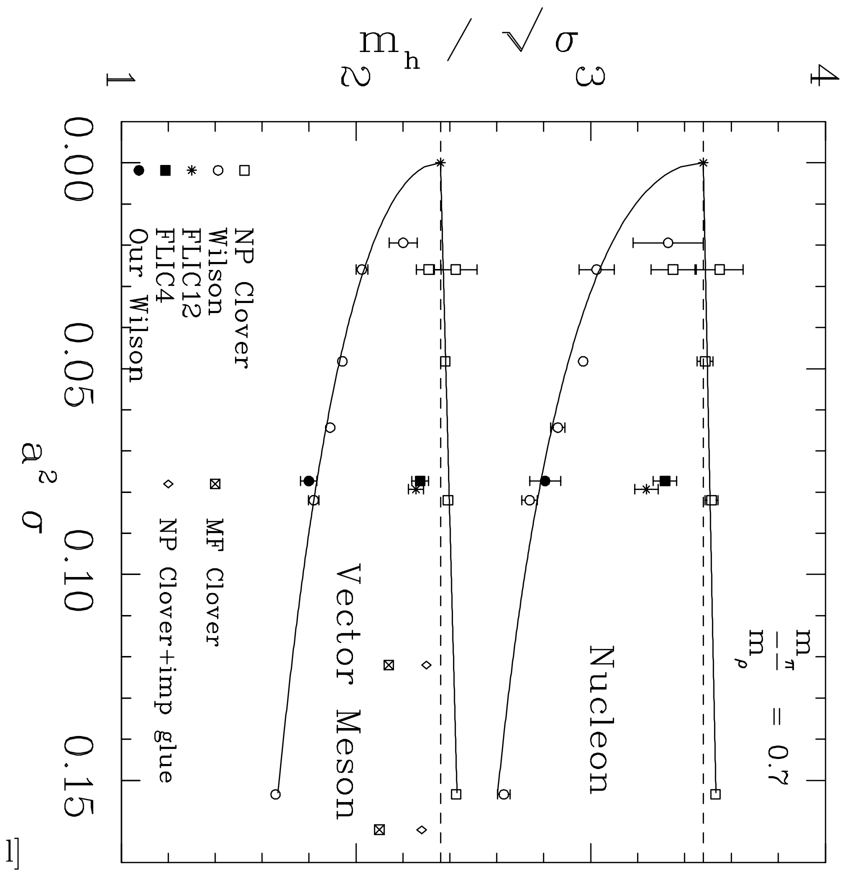

FIG. 3. Nucleon and vector meson masses for the Wilson, NP-improved and FLIC actions obtained by interpolating our results of Fig. 2 to $m_\pi/m_\rho = 0.7$. Results from the present simulations are indicated by the solid points. The fat links are constructed with $n = 4$ (solid squares) and $n = 12$ (stars) smearing sweeps at $\alpha = 0.7$.

simulation results at $a^2\sigma \sim 0.08$, the simulation results that are available indicate that the fat-link results also compete well with those obtained with a NP-improved clover fermion action.

Having determined FLIC4 is the preferred action, we have increased the number of configurations to 200 for this action. As expected, the error bars are halved and the central values for the FLIC4 points move to the upper end of the error bars in Fig. 3, further supporting the promise of excellent scaling.

Figure 4 summarises the entire light hadron spectrum calculated at a quark mass where $m_\pi/m_\rho = 0.7$ for the three actions. It is encouraging to see the correct mass ordering arising in all of the actions. Note that the Wilson action results differ significantly. This is attributed to the large $\mathcal{O}(a)$ errors in the Wilson case. The results also indicate the Δ baryon offers the strongest scaling test, and we will study scaling for this baryon more extensively in future investigations.

Finally, referring back to Table II, we compare the convergence rates of the different

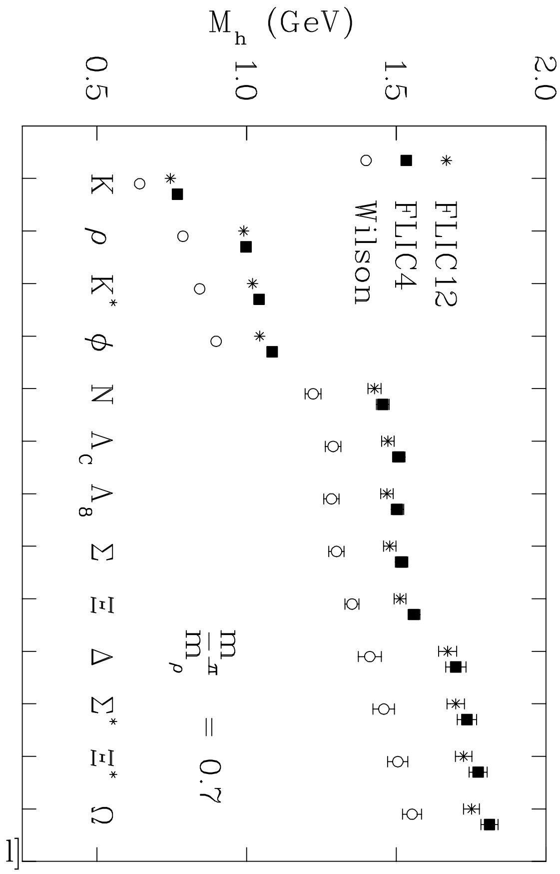

FIG. 4. Light hadron spectrum calculated at $m_\pi/m_\rho = 0.7$ for the Wilson action (circles), FLIC action with $n = 4$ (squares) and $n = 12$ (stars) sweeps.

actions. The FLIC12 action appears to converge at a similar rate to the Wilson fermion action, although it must be pointed out that these are average iterations per mass, and that while the FLIC12 action had most of its convergence iterations clustered closely around these average values, the Wilson action had larger fluctuations around the mean values. However, the FLIC4 action is the clear winner due the small number of BiCGStab iterations required to invert the fermion matrix. This provides great promise for performing simulations at quark masses closer to the physical values.

V. CONCLUSIONS

We have examined the hadron mass spectrum using a novel Fat-Link Irrelevant Clover (FLIC) fermion action, in which only the irrelevant, higher-dimension operators involve smeared links. One of the main conclusions of this work is that the use of fat links in the irrelevant operators provides excellent results. Fat links promise improved scaling behavior over mean-field improvement. This technique also solves a significant problem with $\mathcal{O}(a)$ nonperturbative improvement on mean field-improved gluon configurations. Simulations are possible and the results are competitive with nonperturbative-improved clover results on plaquette-action gluon configurations. We have found that minimal smearing holds the promise of better scaling behavior. Our results suggest that too much smearing removes relevant information from the gauge fields, leading to a poorer performance. Fermion matrix inversion for FLIC4 is more efficient and results show no sign of exceptional configuration problems.

This work paves the way for promising future studies. It will be of great interest to consider different lattice spacings to further test the scaling of the fat-link actions. Furthermore, the exceptional configuration issue can be explored by pushing the quark mass down to lower values. The $n = 4$ FLIC action holds great promise for circumventing this issue as evidenced by the relative ease with which one can invert the fermion matrix (see Table II). A study of the spectrum of excited hadrons using the fat-link clover actions is currently in progress [18].

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